Exchange diagrams in the theory of nuclear matter

1969 J. Phys. A: Gen. Phys. 2 246

(http://iopscience.iop.org/0022-3689/2/2/016)

BROWN, W. D., 1966, *Boeing Rep.*, D2-125136-1.
LAKSHMINARAYANA, V., 1960, *D.Sc. Thesis*, Andhra University.
LAKSHMINARAYANA, V., and JNANANANDA, S., 1961 a, *J. Sci. Res.*, **20B**, 1.
—— 1961 b, *Proc. Phys. Soc.*, **77**, 593.
MCMASTER, W. A., *et al.*, 1967, *UCRL Rep.*, 50174-II.
PLECHATY, E., 1968, *Photon Attenuation Coefficients* (compilation), Lawrence Radiation Laboratory.
SCHMICKLEY, R. D., and PRATT, R. H., 1967, *Phys. Rev.*, **164**, 104.
WYARD, S. J., 1953, *Proc. Phys. Soc.* A, **66**, 382.

J. PHYS. A (GEN. PHYS.), 1969, SER. 2, VOL. 2. PRINTED IN GREAT BRITAIN

Exchange diagrams in the theory of nuclear matter

Abstract. It is shown that the result of including all the exchange diagrams in the expression for the binding energy due to four-body correlations in nuclear matter is to reduce the binding energy due to the direct interaction by a factor of about 3/32.

Bethe (1965) has shown that the correct expansion parameter for the ground-state energy of nuclear matter is essentially the density rather than the g matrix. Thus the significant factor has become the number of interacting nucleons rather than the number of g interactions. Kirson (1967) and Sprung and Bhargava (1967) have done calculations within this framework including two- and three-body correlations. Lawson and Sampanthar (1968) have shown how to treat four-body correlations to *all* orders of perturbation theory. This problem has also been treated by Kuriyama (1968). In this letter we show how the inclusion of all exchange diagrams reduces the effect of the four-body direct terms by a factor of about 3/32.

We assume, as was done first by Rajaraman (1963), the following:

(i) The two-nucleon potential does not depend on spin or i-spin.

(ii) The momentum of the hole states is small, i.e. $k_f c \ll 1$ where k_f is the Fermi momentum and c the radius of the repulsive core in the two-nucleon interaction.

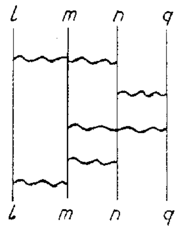

Figure 1. A four-body direct diagram.

Let us consider a connected diagram, shown in figure 1, involving four nucleons. This is a direct diagram since the four nucleons start and end in the same states l, m, n and q. As shown by Kirson (1967), all possible exchange diagrams corresponding to this direct one are obtained by permuting the labels l, m, n and q in the final states of the four nucleons. Figure 2 is an exchange diagram since the final states of the particles 1, 2, 3 and 4 are l, m, q and n (in that order). The assumptions of small hole momenta imply that the contribution due to the diagram shown in figure 2 is essentially the same as that due to the diagram shown in figure 1, except for a minus sign, because of the one exchange of momenta between nucleons 3 and 4.

Further, for this exchange to be possible particles 3 and 4 must have the same spin and i-spin. Thus, if we assign a statistical weight 1 to the contribution due to the diagram of figure 1, in which no restrictions on spin or i-spin exist, we see that we must assign a

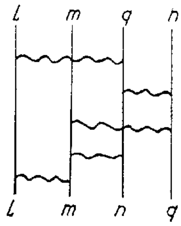

Figure 2. An exchange diagram corresponding to the direct diagram of figure 1.

statistical weight of $-\frac{1}{4}$ to the diagram of figure 2. In this way we can assign statistical weights to all other exchange diagrams. When we consider four-body correlations the following possibilities exist:

(i) Three particles return to their initial states. Then the fourth particle has necessarily to return to its initial state and we get the direct term.

(ii) Two particles return to their initial states. Then the other two particles must exchange momenta.

(iii) One particle returns to its initial state. Then the other three particles must exchange momenta so that none of these returns to its initial state.

(iv) No particle returns to its initial state. There are two cases: (*a*) two particles exchange momenta; the other pair must then follow suit; (*b*) all particles permute amongst themselves, not splitting into pairs.

The numbers of diagrams occurring and their statistical weights are given in table 1, where for completeness the results for two- and three-body correlations are also given.

Table 1

	N	No. of diagrams	Σ-spin, i-spin	Factor	Total/Direct
two-particle correlation	1	1	4^2	+16	
	0	1	4	−4	3/4
three-particle correlation	2	1	4^3	+64	
	1	3	4^2	−48	
	0	2	4	+8	3/8
four-particle correlation	3	1	4^4	+256	
	2	6	4^3	−384	
	1	8	4^2	+128	3/32
	0	(*a*) 3	4^2	+48	
		(*b*) 6	4	−24	

N stands for the number of particles which return to their initial state.

We see that the inclusion of exchange diagrams alters the effect of two-, three- and four-body correlations by factors of 3/4, 3/8 and 3/32 respectively. The smallness of the factor 3/32 indicates further that four-body correlations are relatively unimportant for nuclear matter.

We are grateful to Professor G. E. Brown for correcting an error in an earlier version of this note.

One of us (P.A.L.) would like to acknowledge a Research Studentship from the Science Research Council.

Department of Mathematics,
University of Salford,
Lancs.

P. A. Lawson
S. Sampanthar

9th December 1968, *in revised form 22nd January* 1969

Bethe, H. A., 1965, *Phys. Rev.*, **138,** B804–22.
Kirson, M. W., 1967, *Nucl. Phys.* A, **99,** 353–410.
Kuriyama, A., 1968, *Prog. Theor. Phys., Japan*, **40,** 301–16.
Lawson, P. A., and Sampanthar, S., 1968, *Nucl. Phys.* A, **106,** 669–77.
Rajaraman, R., 1963, *Phys. Rev.*, **129,** 265–70, **131,** 1244–8.
Sprung, D. W. L., and Bhargava, P. C., 1967, *Ann. Phys., N.Y.*, **42,** 222–60.